BEE 233 Circuits
Fall 2015
Lab 4: Opamp filters instructor's notes

4 Circuits

These are the five opamp circuits in this lab. In all cases, the opamp is the same LM348 you worked with in Lab 3, VCC = +12 V and VEE = –12 V.

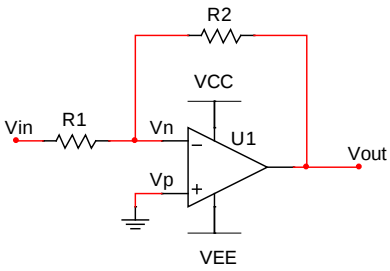


Figure 1. Inverting amplifier.

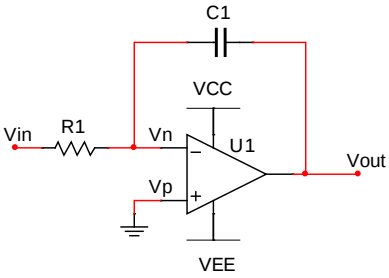


Figure 2. Simple integrator.

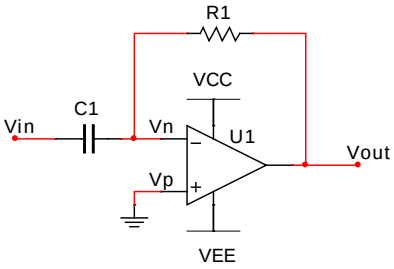


Figure 3. Simple differentiator.

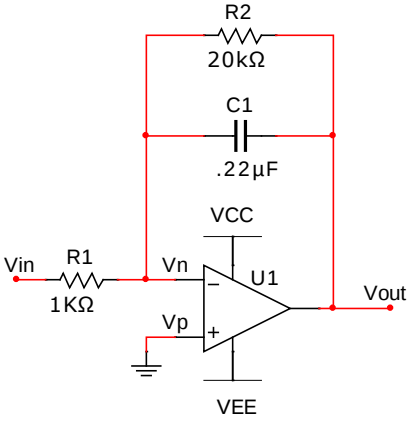


Figure 4. Integrator with shunt.

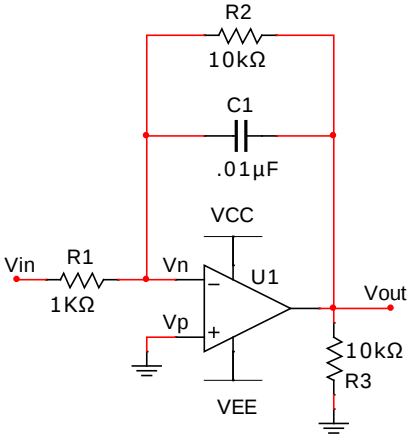


Figure 5. Low-pass filter.

5 Inverting amplifier

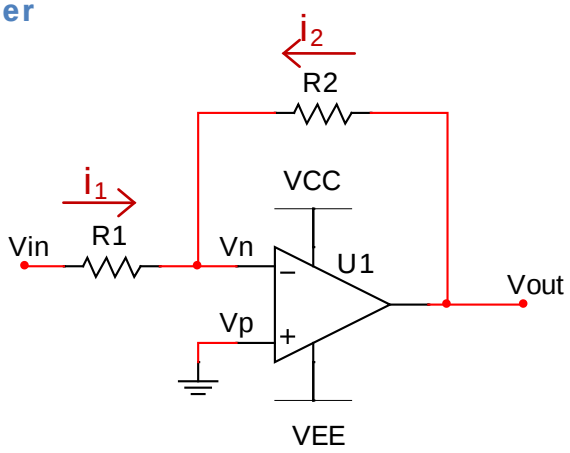


Figure 12. Inverting amplifier.

1. Analyze the inverting amplifier circuit in figure 12 (same as figure 1) to derive an equation for the gain, Av, as a function of R1 and R2.

$$Vn = Vp = 0$$

$$i1 = -i2$$

$$Vin = i1\ R1 + Vn = i1\ R1$$

$$Vout = i2\ R2 + Vn = i2\ R2 = \ -i1\ R2$$

$$Av = \frac{Vout}{Vin} = -\frac{i1\ R2}{i1\ R1} = -\frac{R2}{R1}$$

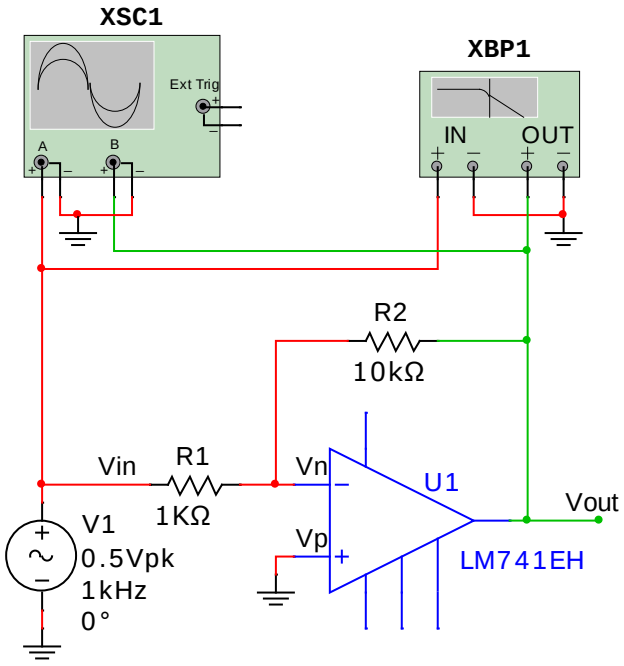
2. Choose values for R1 and R2 to produce a gain, Av = -10.

   For Av1 = –10 and R1 = 1 KΩ, R2 = 10 KΩ

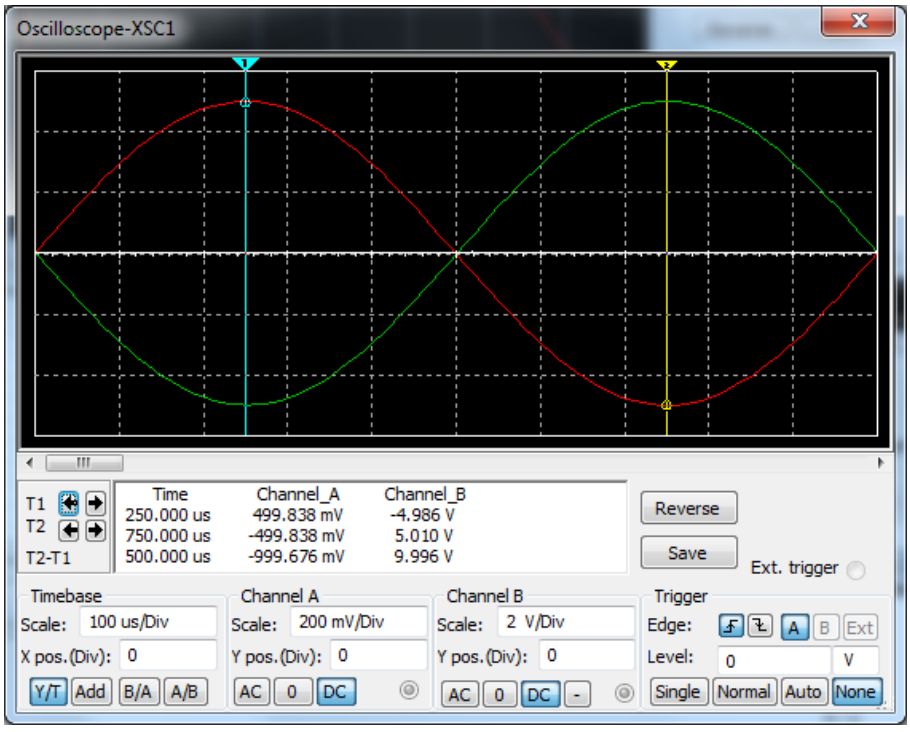
3. Simulate your circuit. Capture images of the following.

   a. Your schematic.

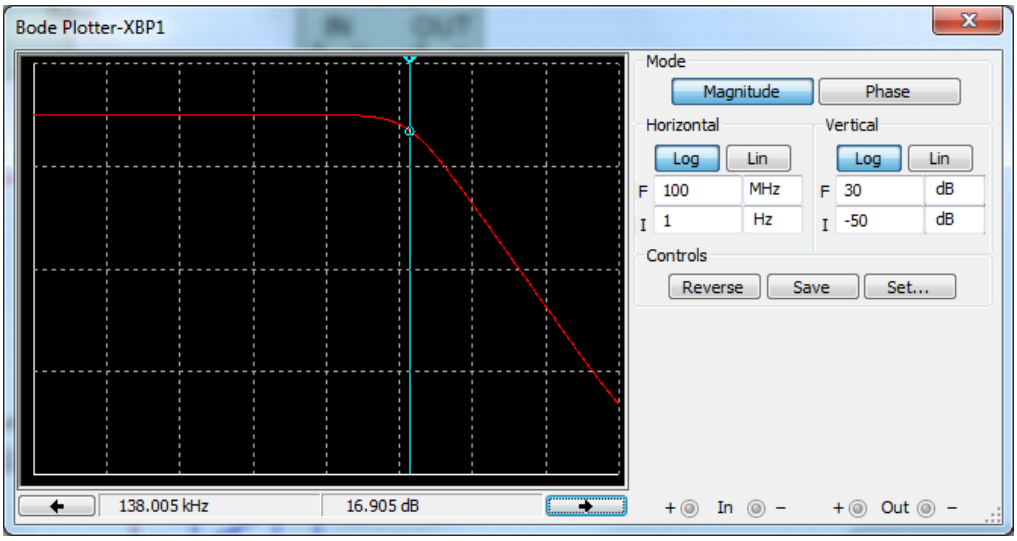
      You should have built something like this and gotten results similar to the following.



   b. An oscilloscope display of Vin and Vout for Vin = 1.0 Vpp, 1 KHz sine wave, 0 V DC offset.



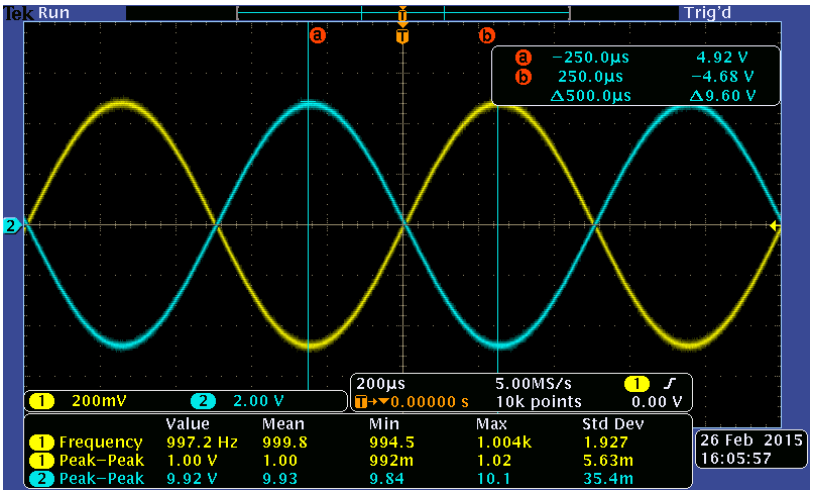
c. A Bode plot of the frequency response with a cursor at the cutoff frequency.

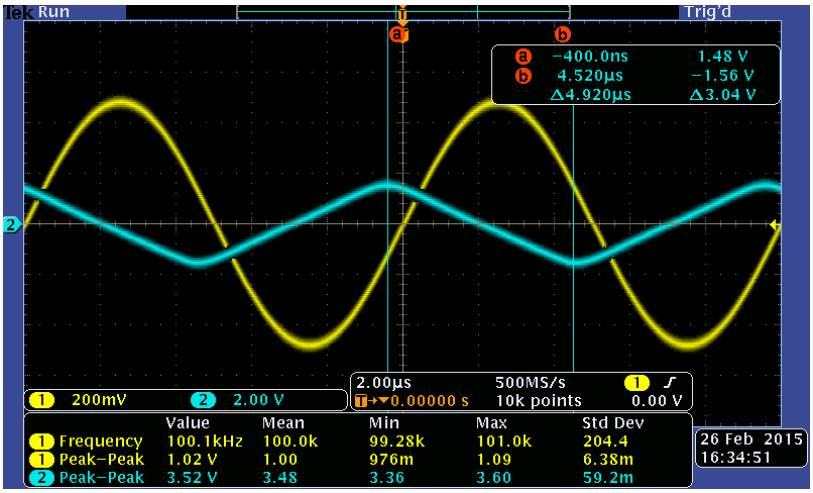


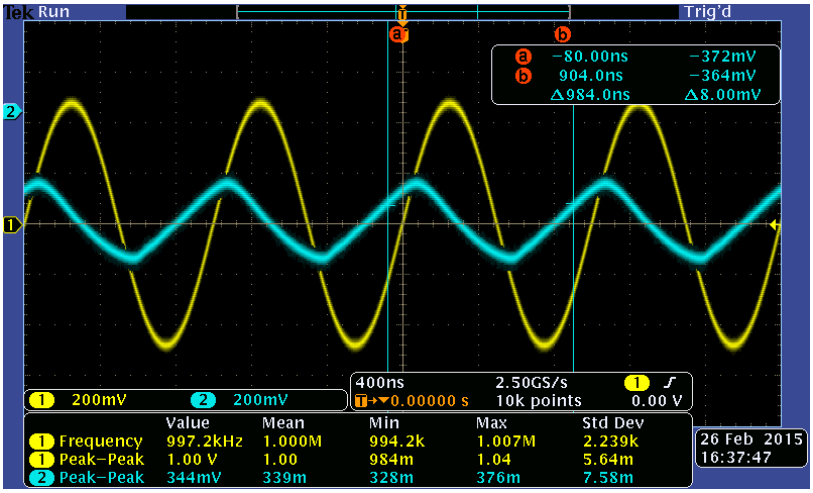
Cutoff was at about 138 KHz in the model that came with my copy of Multisim.

4. Breadboard the circuit and capture screenshots with Vin = 1.0 Vpp sine wave, 0 V DC offset, at 1 KHz, 100 KHz and 1 MHz.

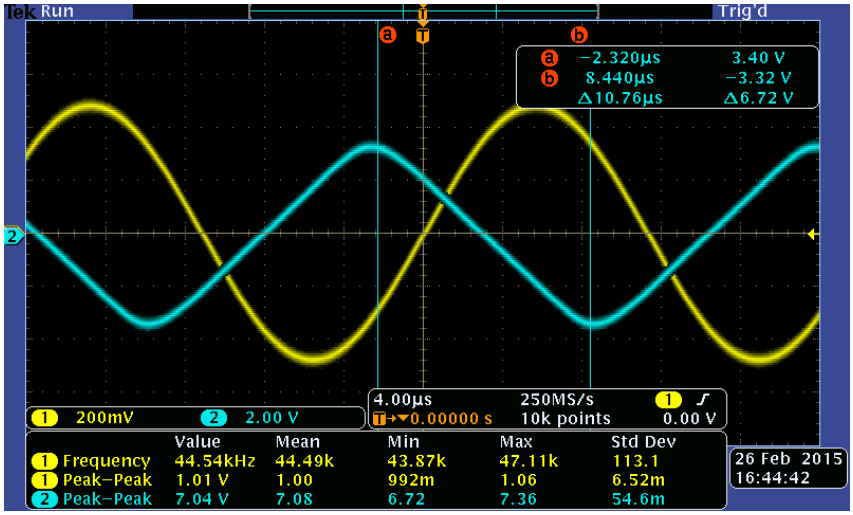
You probably got results about like this. I should have asked for a smaller value of Vin to reduce the effect of hitting slew rate limits.







5. Find the cutoff frequency at which gain has dropped by 3 dB and capture a screenshot.



6. Create a table of measurements at increasing frequencies from 100 Hz to 1 MHz in a 1-2-5-10 sequence. Try to hold Vin fairly constant as you increase the frequency.

Frequency (Hz)	Vin	Vout	Gain (Av)	Gain (dB)
100				
200	Your data goes here			
500				
1K				
2K				
:				
500K				
1M				

7. Create a Bode plot of Gain(dB) versus frequency from your measurements.

   Your plot goes here.

8. Compare the Bode plots and cutoff frequencies based on your simulation with your experimental results and explain any differences.

   Your actual cutoff was probably a lot lower than predicted by simulation, perhaps because the model was wrong or doesn't quite match your chip or because it calculated the Bode plot using smaller input signals than we used and avoided slew rate limits. It's also possible your breadboard added additional stray capacitance or inductance not accounted for in the model.

6 Simple integrator

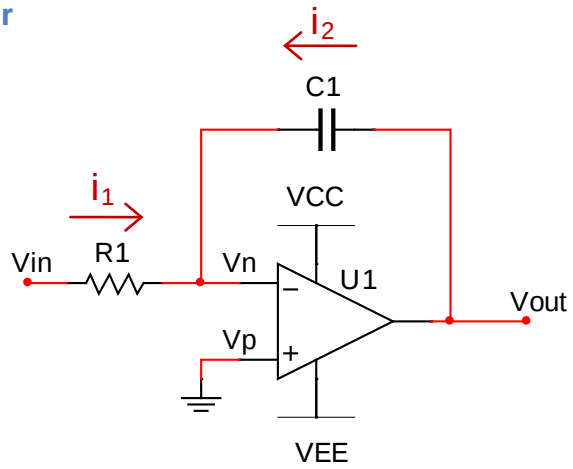


Figure 13. Simple integrator.

1. Analyze the integrator circuit in figure 13 (same as figure 2) to derive an equation for Vout(t) as a function of Vin(t). Show that the output is the integral of the input.

$$Vin = i_1 R_1$$

$$i_1 = \frac{Vin}{R_1} = -i_2$$

$$Vout = \int_0^t \frac{i_2}{C_1} dt$$

$$Vout = -\int_0^t \frac{Vin}{R_1 C_1} dt = -\frac{1}{R_1 C_1} \int_0^t Vin\, dt$$

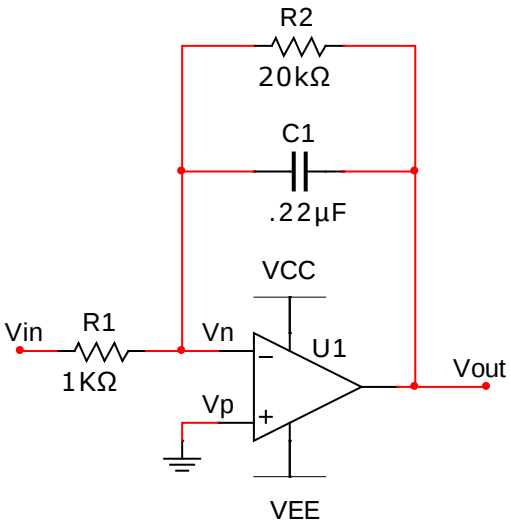


Figure 14. Integrator with shunt.

2. Analyze the circuit in figure 14 (same as figure 4) to calculate the low-frequency gain. Show that for $\gg \frac{1}{R_2 C_1}$, this circuit also functions as integrator.

   All that's required here is a hand-waving argument about what happens at low frequencies, where we can ignore the capacitor, and at high frequencies, where we can ignore the resistor.

   At low frequency, $Z_{C1} \rightarrow \infty$, making it behave like the inverting amp in 6.1.

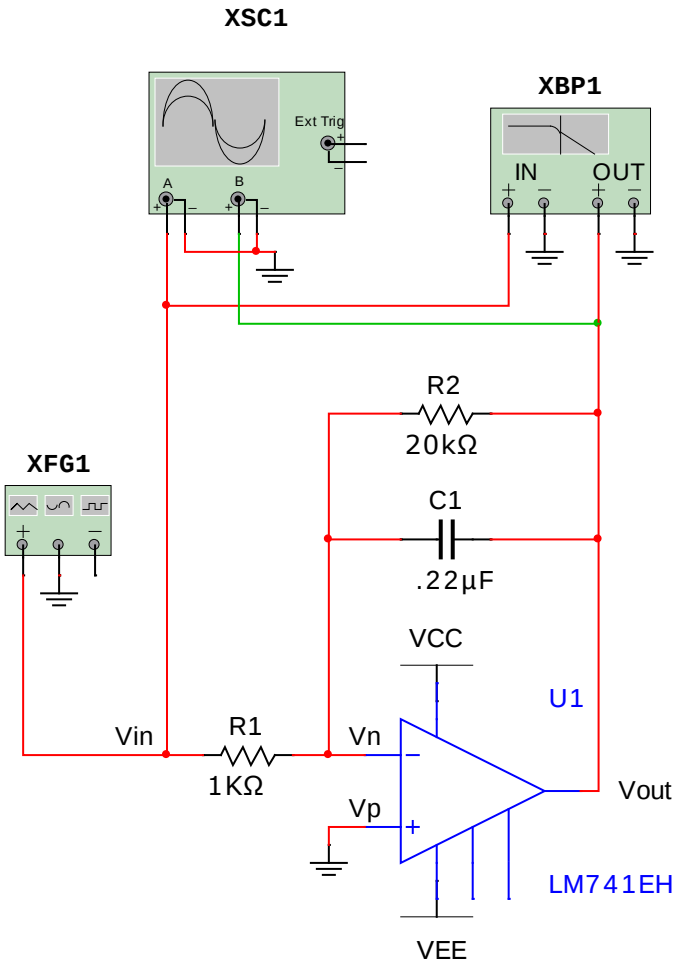
$$Av = -\frac{R2}{R1} = -\frac{20\ K\Omega}{1\ K\Omega} = -20$$

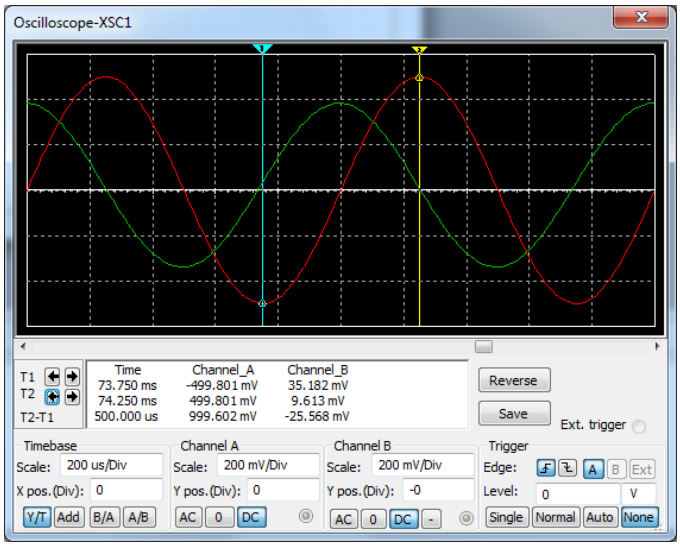
   At high frequency, the impedance of the capacitor is much less than that of R2, making it behave like an integrator.

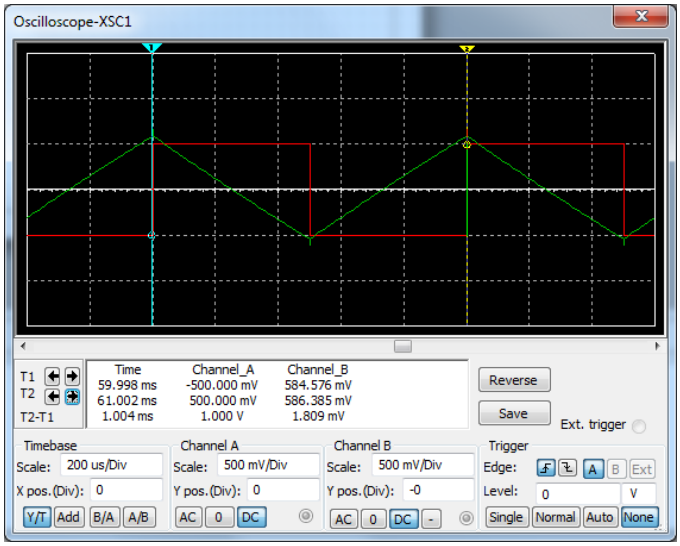
$$Z_{C1} = \frac{1}{sC_1} \ll R_2$$

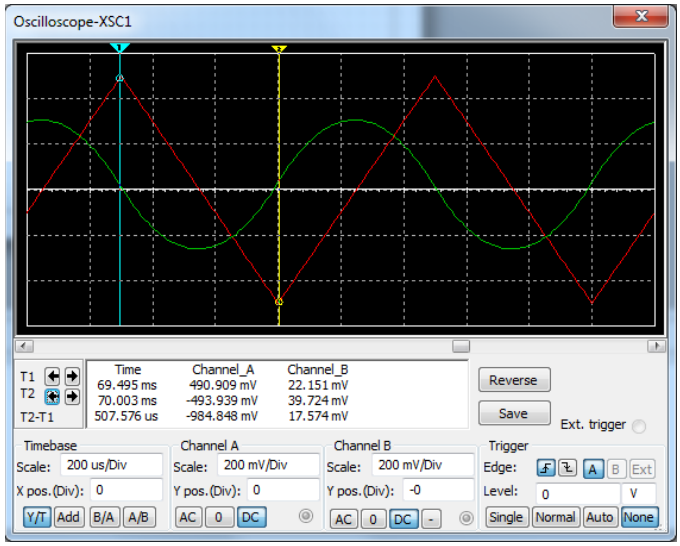
3. Simulate this circuit and capture images of your schematic and of oscilloscope displays of Vin and Vout with Vin set to be a sine wave, a square wave and a triangle wave. In each case, set Vin = 1.0 Vpp at 1 KHz, 0 V DC offset.

   You should have built something like this and gotten results similar to the following for the requested inputs. I found I had to set simulation run time to about .1 s to get it to settle down to a steady state.



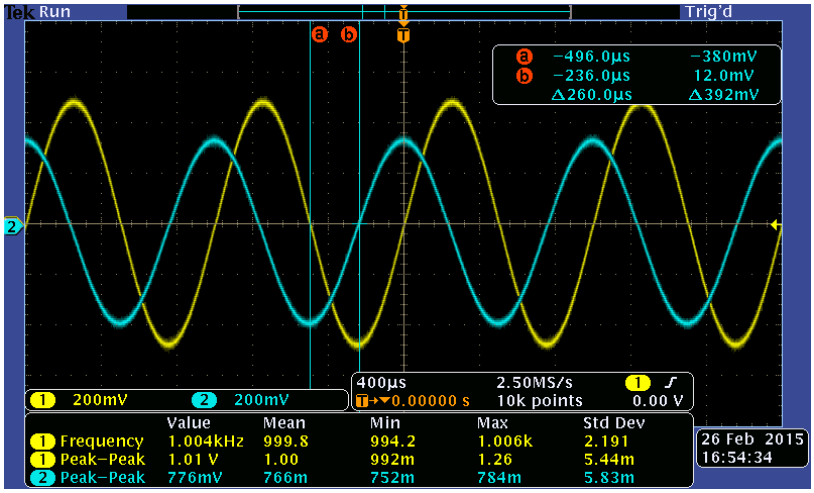

Oscilloscope-XSC1
Time
Channel_A
Channel_B
73.750 ms
-499.801 mV
35.182 mV
74.250 ms
499.801 mV
9.613 mV
500.000 us
999.602 mV
-25.568 mV
T1
T2
T2-T1
Reverse
Save
Ext. trigger
Timebase
Scale: 200 us/Div
X pos.(Div): 0
Y/T Add B/A A/B
Channel A
Scale: 200 mV/Div
Y pos.(Div): 0
AC 0 DC
Channel B
Scale: 200 mV/Div
Y pos.(Div): -0
AC 0 DC -
Trigger
Edge:
A B Ext
Level: 0 V
Single Normal Auto None

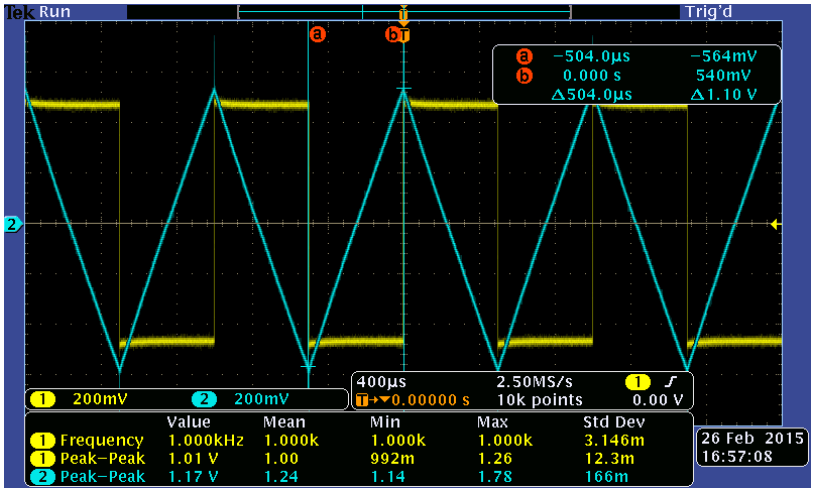

Oscilloscope-XSC1
Time
Channel_A
Channel_B
59.998 ms
-500.000 mV
584.576 mV
61.002 ms
500.000 mV
586.385 mV
1.004 ms
1.000 V
1.809 mV
T1
T2
T2-T1
Reverse
Save
Ext. trigger
Timebase
Scale: 200 us/Div
X pos.(Div): 0
Y/T Add B/A A/B
Channel A
Scale: 500 mV/Div
Y pos.(Div): 0
AC 0 DC
Channel B
Scale: 500 mV/Div
Y pos.(Div): -0
AC 0 DC -
Trigger
Edge:
A B Ext
Level: 0 V
Single Normal Auto None

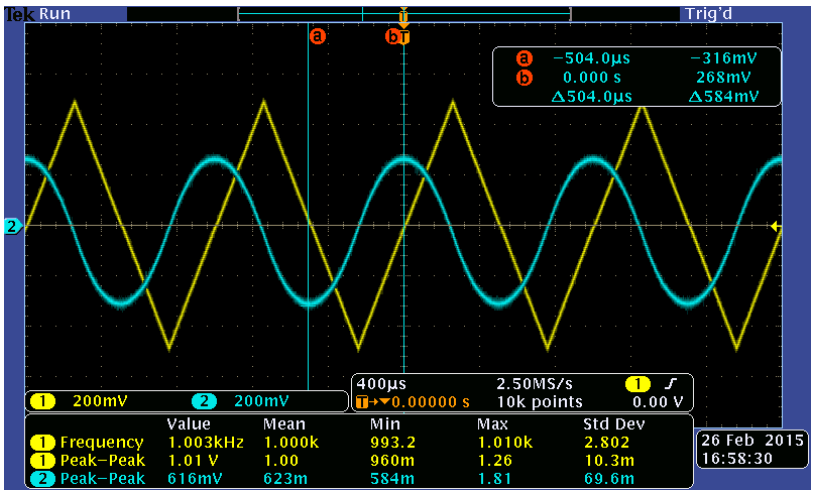

Oscilloscope-XSC1
Time
Channel_A
Channel_B
69.495 ms
490.909 mV
22.151 mV
70.003 ms
-493.939 mV
39.724 mV
507.576 us
-984.848 mV
17.574 mV
T1
T2
T2-T1
Reverse
Save
Ext. trigger
Timebase
Scale: 200 us/Div
X pos.(Div): 0
Y/T Add B/A A/B
Channel A
Scale: 200 mV/Div
Y pos.(Div): 0
AC 0 DC
Channel B
Scale: 200 mV/Div
Y pos.(Div): -0
AC 0 DC -
Trigger
Edge:
A B Ext
Level: 0 V
Single Normal Auto None

4. Build this circuit and capture screenshots with Vin set to be a sine wave, a square wave and a triangle wave = 1.0 Vpp at 1 KHz, 0 V DC offset.

   Your results should have been similar to these.







5. Reset Vin = 0.5 Vpp sine wave at 1 KHz, 0 V DC offset. While it's running, watch the oscilloscope as you remove R2 and then put it back. Describe what you observed. What is the purpose of R2?

   Vout quickly dives either toward VCC or VEE. This happen because $\int_0^t a dt = at$; any tiny DC offset will eventually integrate over time into an arbitrarily large value, limited only by the power supply. The purpose of R2 is to bleed that off faster than it accumulates.

7 Simple differentiator

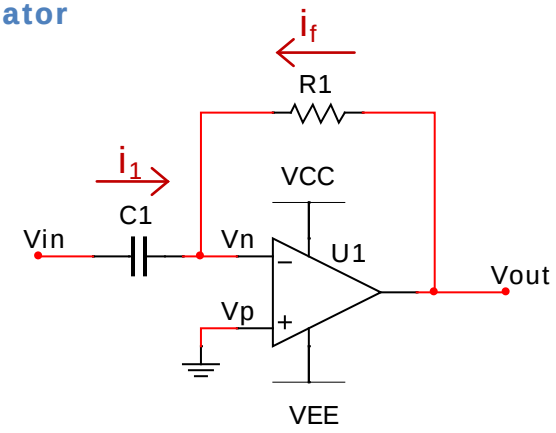


Figure 15. Simple differentiator.

1. Analyze the differentiator circuit in figure 15 (same as figure 3) to derive an equation for Vout(t) as a function of Vin(t). Show that the output is the derivative of the input.

$$Vout = i_f R_1$$

$$i_f = \frac{Vout}{R_1} = -i_1$$

$$Vin = \int_0^t \frac{i_1}{C_1} dt$$

$$Vin\, C_1 = \int_0^t i_1\, dt$$

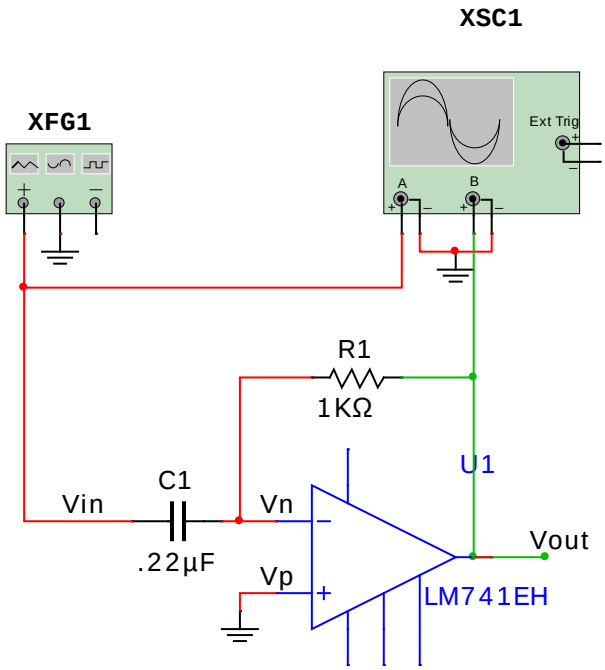
$$i_2 = C_1 \frac{dVin}{dt}$$

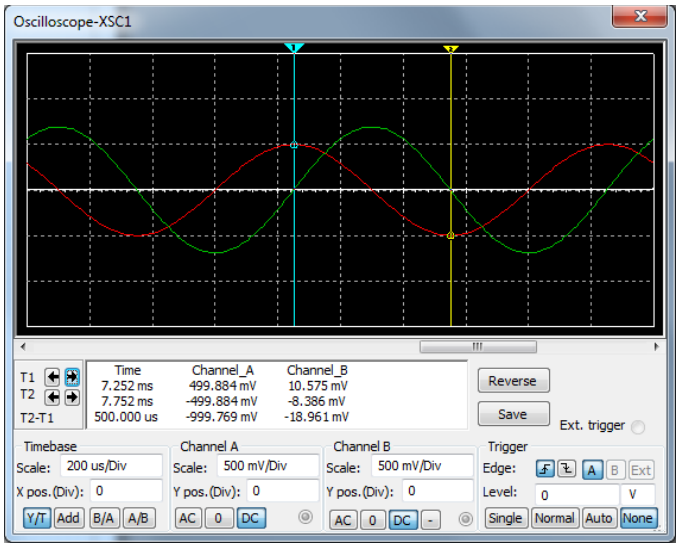
$$i_1 = C_1 \frac{dVin}{dt} = -\frac{Vout}{R_1}$$

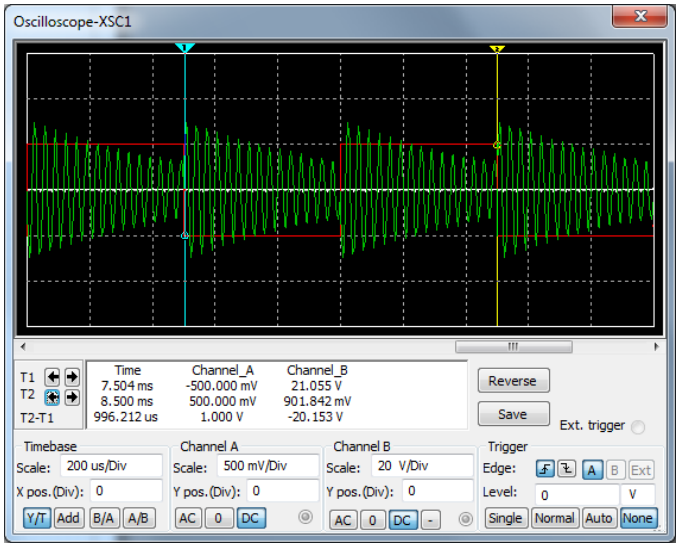
$$Vout = -R_1 C_1 \frac{dVin}{dt}$$

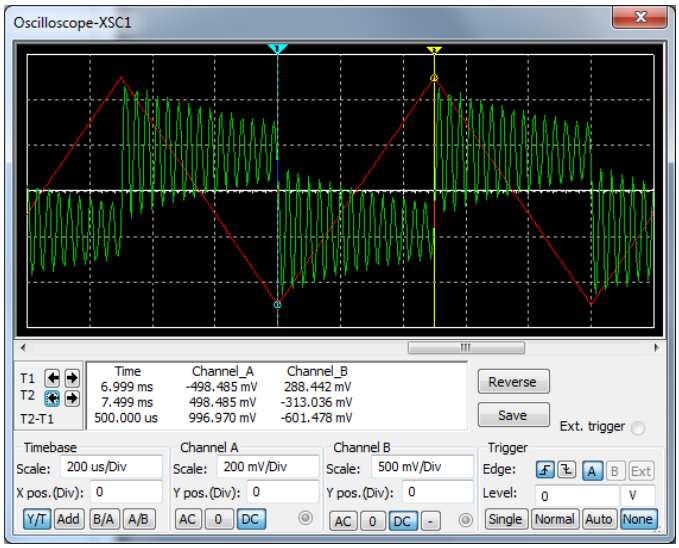
2. Simulate this circuit with R1 = 1 KΩ and C1 = 0.22 µF and capture images of your schematic and of oscilloscope displays of Vin and Vout with Vin set to be a sine wave, a square wave and a triangle wave. In each case, set Vin = 1.0 Vpp at 1 KHz, 0 V DC offset.

   You should have built something like this and may have gotten results similar to the following, predicting that the circuit will work for sine wave input but exhibit severe "ringing" (oscillating) for square and triangle wave inputs.

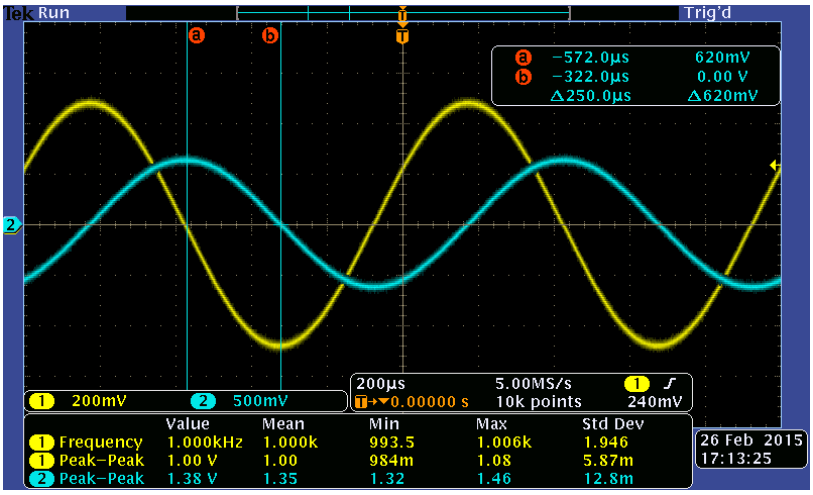


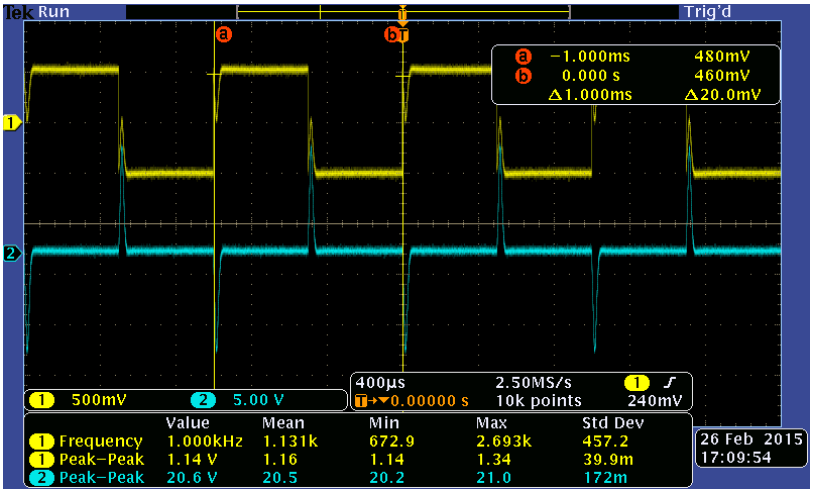

Oscilloscope-XSC1
Time Channel_A Channel_B
T1 7.252 ms 499.884 mV 10.575 mV
T2 7.752 ms -499.884 mV -8.386 mV
T2-T1 500.000 us -999.769 mV -18.961 mV
Reverse
Save
Ext. trigger
Timebase
Scale: 200 us/Div
X pos.(Div): 0
Y/T Add B/A A/B
Channel A
Scale: 500 mV/Div
Y pos.(Div): 0
AC 0 DC
Channel B
Scale: 500 mV/Div
Y pos.(Div): 0
AC 0 DC -
Trigger
Edge: A B Ext
Level: 0 V
Single Normal Auto None

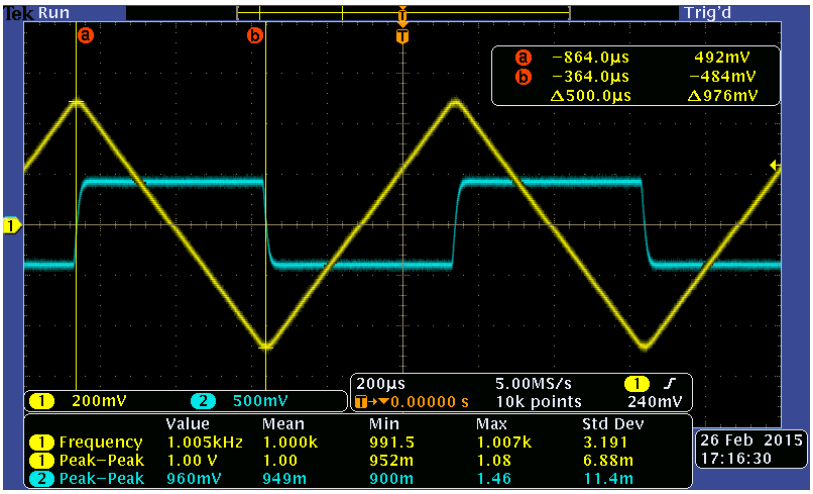

Oscilloscope-XSC1
Time Channel_A Channel_B
T1 7.504 ms -500.000 mV 21.055 V
T2 8.500 ms 500.000 mV 901.842 mV
T2-T1 996.212 us 1.000 V -20.153 V
Reverse
Save
Ext. trigger
Timebase
Scale: 200 us/Div
X pos.(Div): 0
Y/T Add B/A A/B
Channel A
Scale: 500 mV/Div
Y pos.(Div): 0
AC 0 DC
Channel B
Scale: 20 V/Div
Y pos.(Div): 0
AC 0 DC -
Trigger
Edge: A B Ext
Level: 0 V
Single Normal Auto None


Oscilloscope-XSC1
Time Channel_A Channel_B
T1 6.999 ms -498.485 mV 288.442 mV
T2 7.499 ms 498.485 mV -313.036 mV
T2-T1 500.000 us 996.970 mV -601.478 mV
Reverse
Save
Ext. trigger
Timebase
Scale: 200 us/Div
X pos.(Div): 0
Y/T Add B/A A/B
Channel A
Scale: 200 mV/Div
Y pos.(Div): 0
AC 0 DC
Channel B
Scale: 500 mV/Div
Y pos.(Div): 0
AC 0 DC -
Trigger
Edge: A B Ext
Level: 0 V
Single Normal Auto None

3. Build this circuit with R1 = 1 KΩ and C1 = 0.22 μF and capture screenshots with Vin set to be a sine wave, a square wave and a triangle wave = 1.0 Vpp at 1 KHz, 0 V DC offset.







4. Describe what you observe. Does the circuit work as you expected? Can you explain the behavior?

   The real circuit works, behaving the way the math predicts and does not exhibit the ringing predicted by the model. The only noticeable artifact is the blip on Vin when the input is a square wave, caused by the output pulse feeding back through C1. Apparently, the Multisim model is wrong. (Surprise. It's software.)

8 Low pass filter

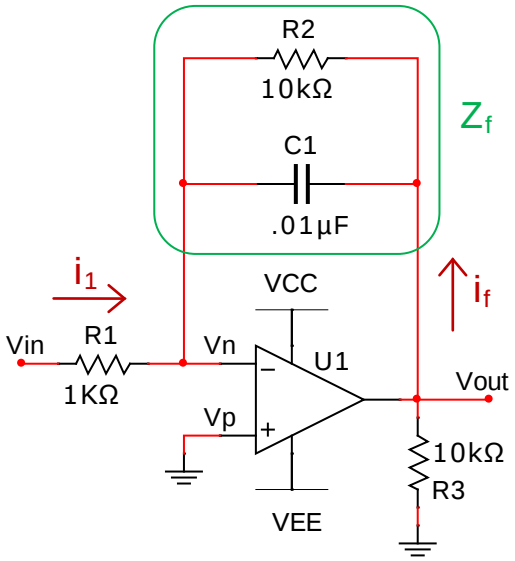


Figure 16. Low-pass filter.

1. Analyze the low-pass filter circuit in figure 16 (same as figure 5) to show that for $Vin(t) = Acos(\omega t)$:

$$Vout(t) = -\frac{R_2}{R_1} Acos(\omega t)\left(\frac{1 - j\omega R_2 C_1}{1 + \omega^2 {R_2}^2 {C_1}^2}\right)$$

$$Y_f = \frac{1}{R_2} + sC_1$$

$$Z_f = \frac{1}{Y_f} = \frac{1}{\frac{1}{R_2} + sC_1} = \frac{R_2}{1 + sR_2C_1}$$

$$i_f = \frac{Vout}{Z_f} = Vout\left(\frac{1 + sR_2C_1}{R_2}\right)$$

$$i_1 = \frac{Vin}{R_1}$$

$$i_f = -i_1$$

$$\frac{Vin}{R_1} = -Vout\left(\frac{1 + sR_2C_1}{R_2}\right)$$

$$\frac{Vout}{Vin} = -\frac{R_2}{R_1}\left(\frac{1}{1 + sR_2C_1}\right)$$

$$Vin = A\cos(\omega t)$$

$$s = j\omega$$

$$\frac{Vout}{Vin} = -\frac{R_2}{R_1}\left(\frac{1}{1 + j\omega R_2C_1}\right)$$

$$\left(\frac{1}{1 + j\omega R_2C_1}\right)\left(\frac{1 - j\omega R_2C_1}{1 - j\omega R_2C_1}\right) = \left(\frac{1 - j\omega R_2C_1}{1 + {R_2}^2{C_1}^2}\right)$$

$$\frac{Vout}{Vin} = -\frac{R_2}{R_1}\left(\frac{1 - j\omega R_2C_1}{1 + \omega^2{R_2}^2{C_1}^2}\right)$$

$$Vout = -\frac{R_2}{R_1}\left(\frac{1 - j\omega R_2C_1}{1 + \omega^2{R_2}^2{C_1}^2}\right)Vin$$

$$Vout = -\frac{R_2}{R_1}A\cos(\omega t)\left(\frac{1 - j\omega R_2C_1}{1 + \omega^2{R_2}^2{C_1}^2}\right)$$

2. Show that for the component values shown, the voltage gain, Av, of this circuit is:

$$Av = \frac{Vout}{Vin} = -\frac{10^5}{\sqrt{10^8 + \omega^2}}$$

$$Av = \frac{Vout}{Vin} = -\frac{R_2}{R_1}\left(\frac{1}{1 + j\omega R_2C_1}\right)$$

$$R_1 = 1K = 10^3$$

$$R_2 = R_3 = 10K = 10^4$$

$$C_1 = 0.01\,\mu F = 10^{-8}$$

$$Av = -\frac{10^4}{10^3}\left(\frac{1}{1 + j\omega(10^4)(10^{-8})}\right) = -10\left(\frac{1}{1 + j\omega(10^{-4})}\right)$$

To get from here to the requested form, you have to notice that $j\omega(10^{-4})$ will be small compared to 1 since this is intended as a low-pass filter, allowing us to approximate with the magnitude of that complex denominator. (I probably should have rewritten this question to ask for |Av|, not Av, and will next time.) Notice that when ω is very small, Av = -10 as expected.

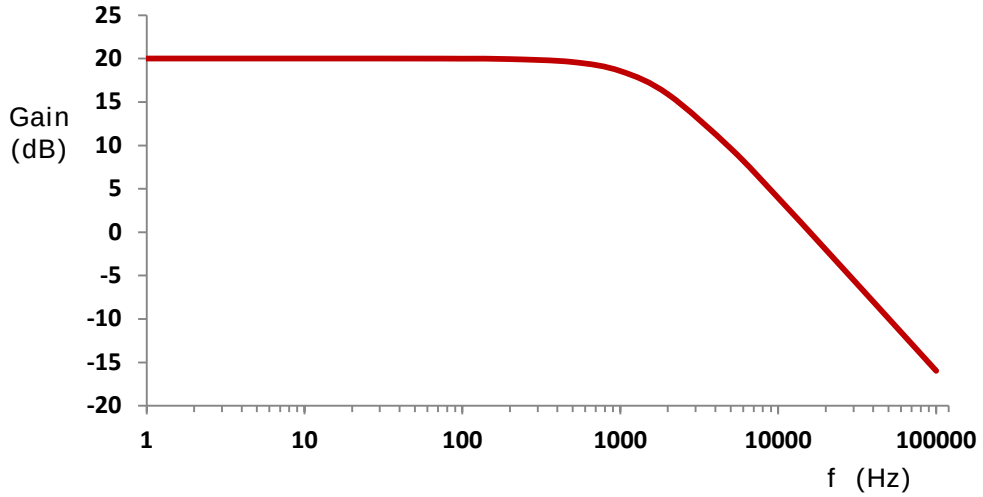
$$|1 + j\omega(10^{-4})| = \sqrt{1 + \omega^2(10^{-8})}$$

$$Av \cong -10\left(\frac{1}{\sqrt{1 + \omega^2(10^{-8})}}\right)$$

To get it into the final requested form, multiply top and bottom by 10^4.

$$Av = -10\left(\frac{1}{\sqrt{1 + \omega^2(10^{-8})}}\right)\left(\frac{10^4}{10^4}\right) = -\frac{10^5}{\sqrt{10^8 + \omega^2}}$$

3. Create a Bode plot of the expected gain from 1 Hz to 100 KHz and calculate the expected cutoff frequency.



Gain at low frequency is 20 dB = 20 log10(|Av|) = 20 log10 (10). The cutoff frequency is the point at which gain has dropped by 3 dB to 17 dB.

$$17 = 20\ \log_{10}(|Av|)$$

$$\frac{17}{20} = \log_{10}(|Av|)$$

$$10^{\frac{17}{20}} = |Av| = 7.079$$

$$\frac{10^5}{\sqrt{10^8 + \omega^2}} = 7.079$$

$$\frac{10^5}{7.079} = \sqrt{10^8 + \omega^2} = 14125.37$$

$$10^8 + \omega^2 = 14125.37^2 = 199526231.5$$

$$\omega^2 = 199526231.5 - 10^8 = 99526231.5$$

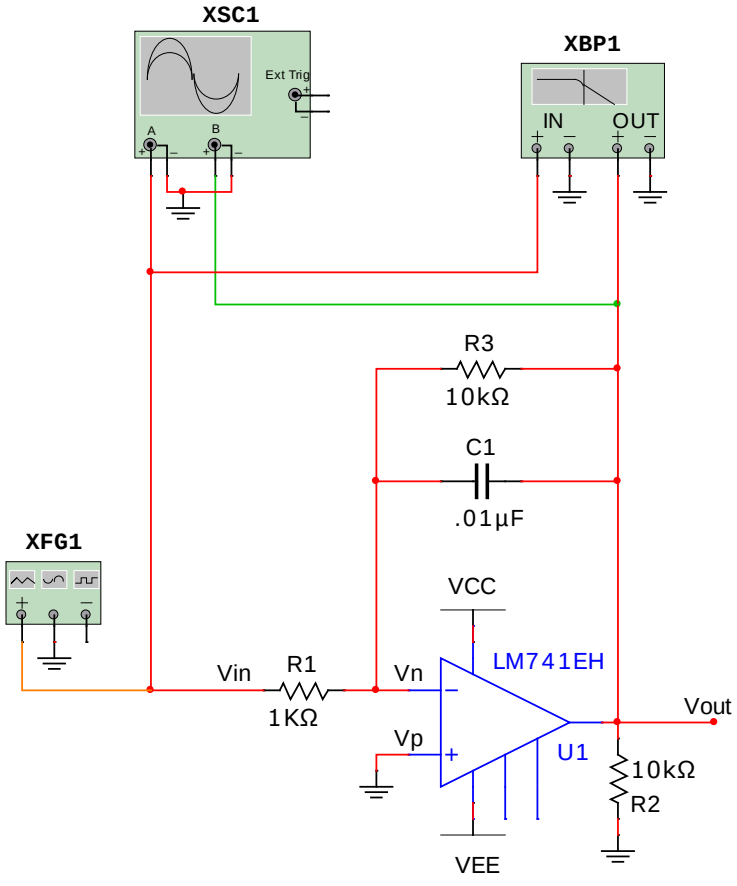
$$\omega = 9976.28 \text{ radians/s}$$

$$f = \frac{\omega}{2\pi} = 1587.77 \text{ Hz}$$

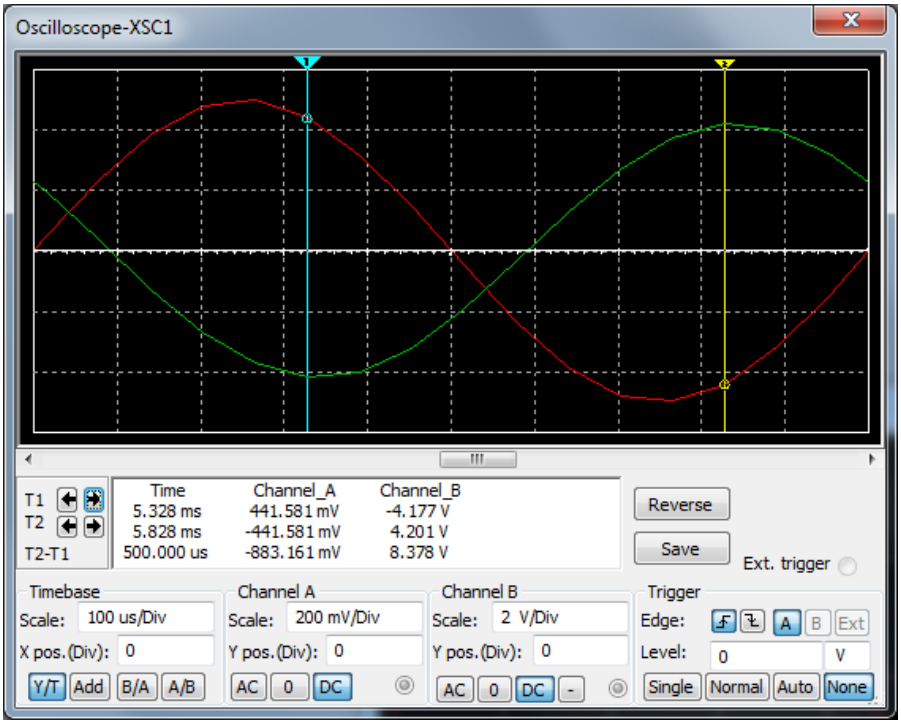
4. Simulate your circuit. Capture images of the following.

   a. Your schematic.

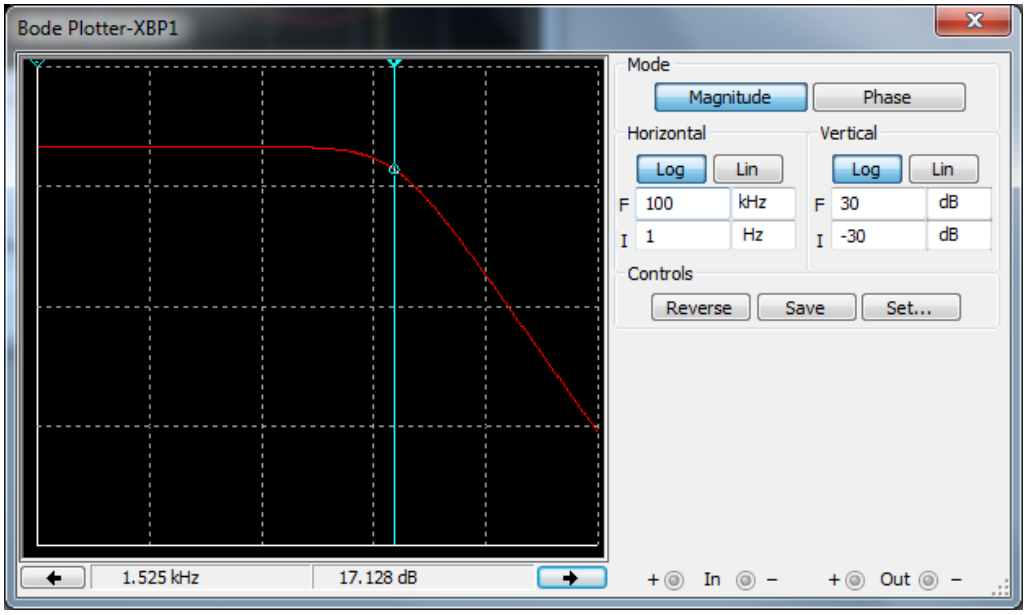
   You should have built something like this and gotten similar results.



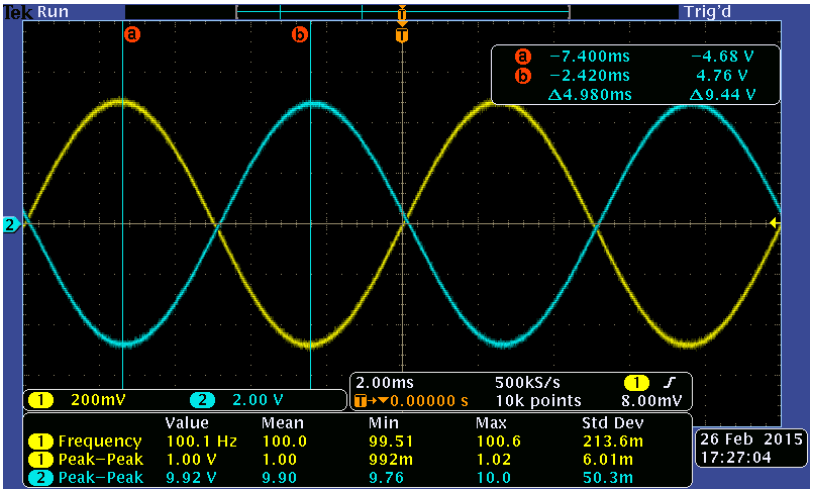
b. An oscilloscope display of Vin and Vout for Vin = 1.0 Vpp, 1 KHz sine wave, 0 V DC offset.

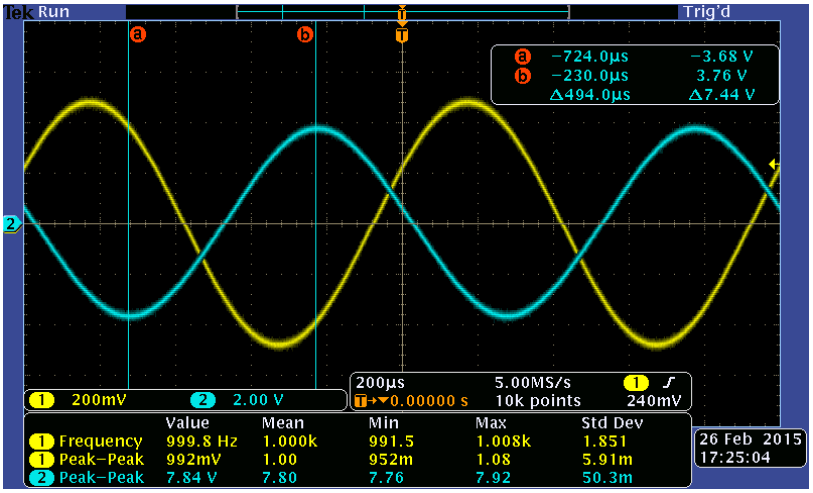


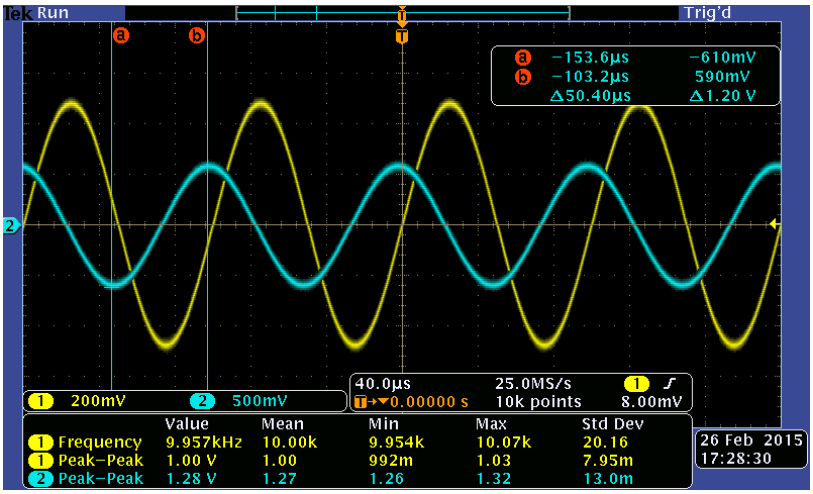
c. A Bode plot of the frequency response with a cursor at the 3 dB point.



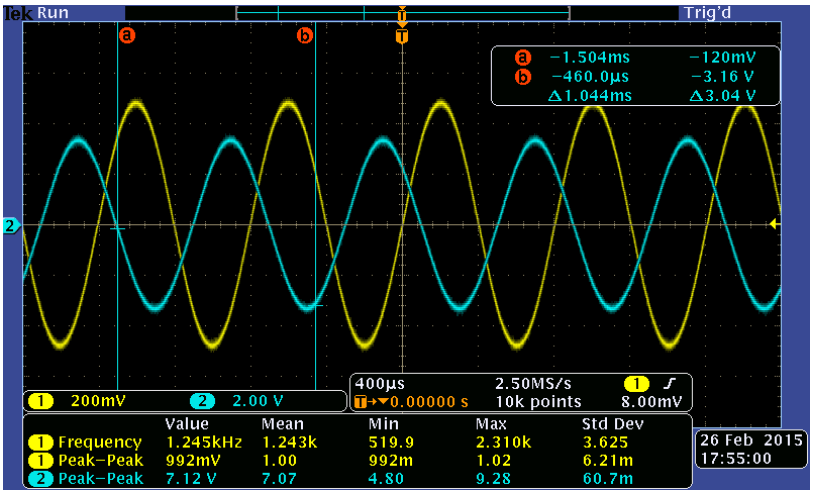
5. Breadboard the circuit and capture screenshots with Vin = 1.0 Vpp sine wave, 0 V DC offset, at 100 Hz, 1 KHz and 10 KHz.







6. Find the frequency at which gain has dropped by 3 dB and capture a screenshot.



7. Create a table of measurements at increasing frequencies from 100 Hz to 100 KHz in a 1-2-5-10 sequence. Try to hold Vin fairly constant as you increase the frequency.

Frequency (Hz)	Vin	Vout	Gain (Av)	Gain (dB)
100				
200	Your data goes here			
500				
1K				
2K				
:				
50K				
100K				

8. Create a Bode plot of Gain(dB) versus frequency from your measurements.

   Your plot goes here.

9. Compare your three Bode plots and cutoff frequencies based on your calculations, your simulation and your experimental results and explain any differences.

   Your calculated and simulated results should have agreed pretty well but it wouldn't be surprising if your measured values are off due to component tolerances. Here were my results.

	Cutoff frequency (Hz)
Calculated	1588
Simulated	1525
Measured	1245